# Geometry at Work

## Summer Mini-Project #1



••••••••Cabinetmaker

**Cabinetmakers** not only make cabinets but all types of wooden furniture. The artistry of cabinetmaking can be seen in the beauty and uniqueness of the finest doors, shelves, and tables. The craft is in knowing which types of wood and tools to use, and how to use them.

The carpenter's square is one of the most useful of the cabinetmaker's tools. It can be applied to a variety of measuring tasks. The figure shows how to use a carpenter's square to bisect  $\angle AOC$ .



#### EXERCISE

Use 8.5x11 or similar sized paper to draw two rays *OA* and *OC* as shown above. Using a carpenter's square (if you can find one), follow the directions in the inset above to draw ray *OB* which will bisect the angle created by *OC* and *OC*. If you don't have a carpenter square, tape 2 rulers together at a right angle (use something square/rectangular to align the two rulers). Then answer the following question:

Is the fact that the carpenter's square uses a right (90°) angle the reason the two new (bisected) angles at O are equal? If not (meaning that they'd also be equal if the carpenter's square was at (say) 75°), explain in your own words why the bisected angles are equal. (Hint: note your drawing with the carpenter square makes two triangles. What do you notice about their sizes/shapes?)

### Summer Mini-Project #2

Much of Manhattan is laid out in a rectangular grid, as shown in this map. In general, the streets are parallel running east and west. The avenues are parallel running north and south.



Yvonne's family is at the corner of 44th Street and 7th Avenue. They plan to walk to Madison Square Park at 23rd Street and 5th Avenue. There are several possible routes they can take.

#### EXERCISES

#### Use tracing paper to trace the routes on the map. Answer the following questions.

1. Yvonne's father wants to walk east on 44th Street until they reach 5th Avenue. He then plans to walk south on 5th Avenue to Madison Square Park. About how long is his route?



- **2.** Yvonne's mother wants to walk south on 7th Avenue until they reach 23rd Street. She then plans to walk east on 23rd Street to Madison Square Park. About how long is her route?
- **3.** Yvonne notices on the map that Broadway cuts across the grid of streets and leads directly to Madison Square Park. She suggests walking all the way on Broadway. About how long is her route?
- 4. Whose route is the shortest? Explain.
- 5. Whose route is the longest? Explain.
- **6.** Make a plausible argument for the following statement: Walking the diagonal street connecting a cross street and an avenue will ALWAYS be shorter than walking the connected street and turning 90° and then walking to the connection on the avenue.
- 7. In your local town, find a section of your streets that are laid out like the numbered streets, the numbered avenues, and Broadway in Manhattan (cross-streets, avenues, and a diagonal street). Using your phones' GPS, measure the length of the triangle's 3 legs. Record the street names and their lengths below: Draw a rough map of where you walked. What can you say about the distance of the diagonal to the two legs?

### Summer Mini-Project #3

Using graph paper, draw any rectangle. Label the sides a and b. Cut four rectangles with length a and width b from the graph paper. Then cut each rectangle on its diagonal, c, forming eight congruent triangles. (If you don't have graph paper, use any paper just being sure the measurement of all eight a as ides are the same length and all eight b sides are the same length and their corners are 90°).





Cut three squares from colored paper, one with sides of length a, one with sides of length b, and one with sides of length c (the diagonal). Again if you don't have colored paper, just use pencil or crayon to distinguish the three boxes.

Separate the eleven pieces into groups.

Group 1	Group 2
four triangles and the two smaller squares	four triangles and the largest square

Arrange the pieces of each group to form a square.

1. a. How do the areas of the two squares you formed in the last step above compare?

Write an algebraic expression for the area of each of these squares.

- (the triangles are called  $\frac{1}{2}a \cdot b$ ),
- the squares are called  $a^2$ ,  $b^2$ , and  $c^2$

b. What can you conclude about the areas of the three squares you cut from colored paper?



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2. Repeat this investigation using a new rectangle with different a and b values. What do you notice?

- 3. Express your conclusion as an algebraic equation (from above).
- **4.** Use your ruler with any rectangle to find actual measures for a,b, and c. Do these measures confirm that  $a^2 + b^2 = c^2$ ?
- **5.** Explain how the diagram at the right represents your conclusion in Exercise 3.



# Summer Mini-Project #4

- Draw and cut out a large triangle.
- Number the angles and tear them off.
- Place the three angles adjacent to each other to form one angle, as shown at the right.
- 1. What kind of angle is formed by the three smaller angles? What is its measure (in degrees)?
- **2.** Make a conjecture about the sum of the measures of the angles of a triangle.

### EXERCISE

- Fold a sheet of paper in half three times. Draw a scalene triangle (no two sides congruent) on the folded paper. Carefully cut out the triangle. This will give you eight triangles that are all the same size and shape.
- Number the angles of each triangle 1, 2, and 3. Use the same number on the corresponding angles from one triangle to the next.
- **3.** Mark a point P in the middle of a blank sheet of paper. How many of your triangles do you think you can fit perfectly about point P without gaps or overlaps?
- **4.** What conjecture can you making about the sum of the angles that encircle a point? What is its measure (in degrees)?









# Geometry at Work

## Summer Mini-Project #4

You can create a three-dimensional space figure with a two-dimensional *perspective drawing*. Suppose two lines are parallel in three dimensions but recede from the viewer. You draw them—and create perspective—so that they meet at a *vanishing point* on a *horizon line*.

## EXAMPLE

Draw a cube in one-point perspective.

 Step 1: Draw a square.
 ✓
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 Then draw a horizon
 Ine and a vanishing
 Ine

 point on the line.
 Ine
 Ine

**Step 2:** Lightly draw segments from the vertices of the square to the vanishing point.



**Step 4:** Complete the figure by using dashes for hidden edges of the cube. Erase unneeded lines.



Two-point perspective involves the use of two vanishing points.

# EXAMPLE

• in Step 2.

Step 3: Draw a square

Each vertex should lie

on a segment you drew

for the back of the cube.

Draw a box in two-point perspective.

**Step 1:** Draw a vertical segment. Then draw a horizon line and two vanishing points on the line.

**Step 2:** Lightly draw segments from the endpoints of the vertical segment to each vanishing point.

**Step 3:** Draw two vertical segments between the segments of Step 2.

**Step 4:** Draw segments from the endpoints of the segments you drew in Step 3 to the vanishing points.

**Step 5:** Complete the figure by using dashes for hidden edges of the figure. Erase unneeded lines.





